Minimum Mean Variance Portfolio of Croatian Stocks

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Abstract

The purpose of this paper is to apply and test the concept of minimal mean variance models to Croatian market. The minimum mean variance models are one of the core ideas behind modern portfolio theory. These types of models are based on a trade-off between risk and return. We are going to apply and test the model using data for Croatian securities. The idea is to form a sensible portfolio using theoretical postulates of minimum mean variance portfolio models. Furthermore, we are going o compare the returns on our portfolio with the market returns. Than we will conclude weather it is possible and sensible to apply this model on small market with short history, as is the case with the Croatian market. The stocks that will be taken into consideration will be chosen according to liquidity criteria. So, we are not including stocks that are traded irregularly, but stocks that have some liquidity and transparency.

Keywords: capital market, portfolio, risk, return
JEL Classification: C29, G11

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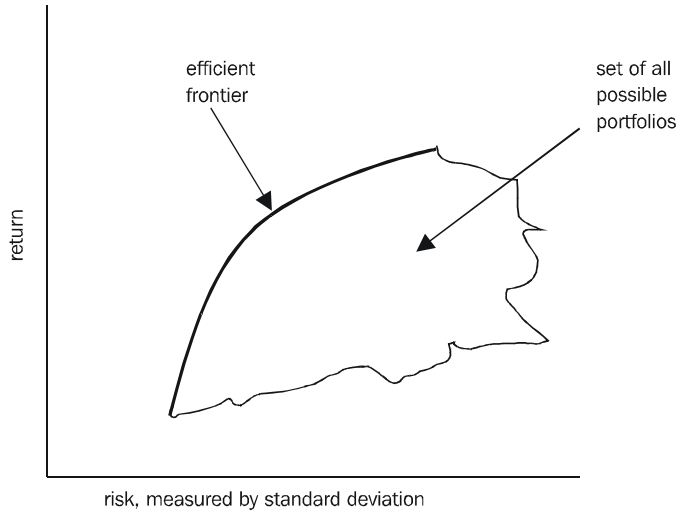
1 Theory Behind the Calculations

Efficient frontier was first discovered by Harry Markowitz in Portfolio selection (Markovitz, 1952). The portfolios on an efficient frontier have certain special properties. For a given level of return, the portfolio on the efficient frontier has the lowest possible risk. And vice versa, for a given level of risk portfolio on efficient frontier has the highest possible return.

Figure 1. **Efficient frontier**



Source: Markovitz (1952) adapted by the authors.

The efficient frontier can be found using algorithms used by Elton and Gruber (Elton and Gruber, 1995). We are not going to rewrite them here, but just state their basic theoretical postulates.

In the simplest case, where there is no **constrain on short sales or risk-less lending and borrowing** the idea is to maximize the function:

$\theta = \frac{R_P - R_F}{\sigma_P}$, subject to the constraint $\sum_{i=1}^{n} \chi_i = 1$,

Where: R_P is average return on portfolio

R_F is risk free rate

σ_P is portfolio's variance

x_i weight of asset i in portfolio

and $\sum_{i=1}^{n} x_i R_i = R_P$

By maximizing the function for varying R_F we can trace out the entire frontier. If we do ***not allow short sales***, but still allow risk-less lending and borrowing, there is one additional constraint:

$x_i \geq 0$ for all i.

Our goal for this research is to make optimal risky portfolio from Croatian stocks and to see whether it performed better than market portfolio (we used CROBEX index as market portfolio). Croatian market is very small compared to the developed markets, so it is interesting to see how the portfolio theory applies to Croatia.

2 Stocks Selection Process

We chose the most liquid 18 stocks from both Zagreb and Varaždin Stock Exchange. Due to the Law on public firms, introduced in 2003, companies that have more than 100 shareholders or more than 30 million kuna were forced to list their stocks on the stock exchange. For this reason, some companies that were very interesting to investors became listed. The interest for these stocks can be seen from their trading volumes as well as from rise in stock prices. However, this event also constrained our research because our time series became rather short. Since some of the companies that we chose became listed only in the first half of 2003 we had to take all stock prices from that period in order to calculate correlation and covariance matrix. Still, we had to leave out the stock with the second highest turnover in first half of 2004 - Adris stock - due to short time that this stock has been listed on the Zagreb Stock Exchange. Also we had to leave out Varaždinska bank, because this bank was merged with Zagrebačka bank and its stocks stopped trading on the stock exchanges (1 stock of Zagrebačka bank was exchanged with 8 stocks of Varaždinska bank), Kerametal because there was only one transaction in the first half of 2004 and due to that transaction Kerametal ended in the first ten most traded stocks. Similar to this stock was Maraska, which had only a few big transactions on the Varaždin Stock Exchange. As an approximation of market portfolio

in Croatia we used CROBEX index (the official index of Zagreb Stock Exchange stocks), which we used to compare with our calculated optimal portfolio.

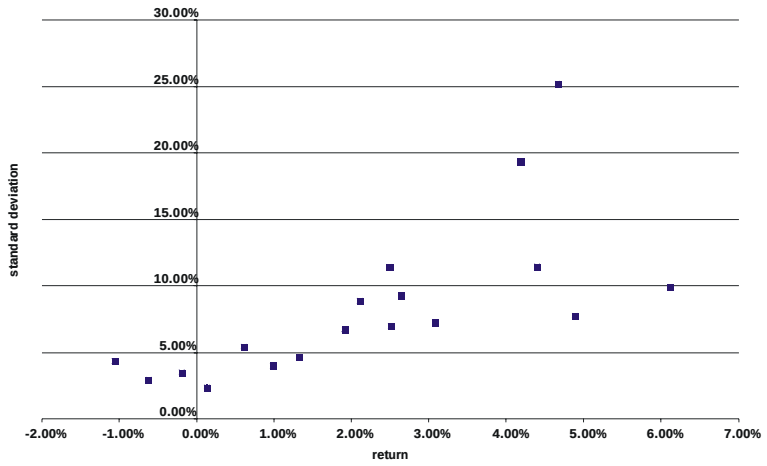
Table 1. **Stocks from Zagreb and Varaždin stock exchange that we used**

Name of the company	Ticker
Croatia osiguranje	CROS-R-A
Elka	ELKA-R-A
Končar	KOEI-R-A
Kraš	KRAS-R-A
Privredna banka Zagreb	PBZ-R-A
Plava laguna	PLAG-R-A
Pliva	PLVA-R-A
Podravka	PODR-R-A
Zagrebačka banka	ZABA-R-A
Sunčani Hvar	SUNH-R-A
Karlovačka banka	KABA-R-A
Rivijera Poreč	RIVP-R-A
Istraturist	ISTT-R-A
Proficio	PRFC-R-A
Dom holding	KORF-R-A
Slavonski ZIF	SLPF-R-A
SN Holding	SNHO-R-A
Ericsson Nikola Tesla	ERNT-R-A

3 Calculation

The data used was for the 16 months from which we calculated average monthly returns, standard deviations, and correlation and covariance matrixes. We took end of the month stock prices from March 2003 until June 2004. In order to get optimal portfolio we needed to chose risk free assets and therefore we chose 3 month Treasury bill from the end of June 2004, which was then 3,95 percent.

Graph 1. **Return versus standard deviation for historical data of 18 stocks**



Next step was finding weights of stocks in optimal portfolio from historical data. Using Mathematica 5.0 we calculated weights for unconstrained optimal portfolio, but since in unconstrained optimal portfolio it is allowed to short sell and long buy stocks, and in Croatia it is not possible to short sell stocks, we decided to use only constrained optimal portfolio in our analysis. We used Excel spreadsheets for this constrained portfolio optimatisation. Optimal weights that we calculated did not pay attention on a discrete number of stocks, so these portfolio weights would serve best for investment funds.

Table 2. **Weights of stocks in the constrained optimal portfolio**

Ticker	Weight
CROS-R-A	14.5%
ELKA-R-A	15.2%
KOEI-R-A	0.9%
KRAS-R-A	1.2%
PBZ-R-A	6.0%
PLAG-R-A	12.7%
PLVA-R-A	2.4%
PODR-R-A	3.2%
ZABA-R-A	0.3%
SUNH-R-A	1.4%
KABA-R-A	0.2%
SNHO-R-A	12.4%
KORF-R-A	0.3%
PRFC-R-A	1.1%
ERNT-R-A	6.6%
SLPF-R-A	12.5%
RIVP-R-A	8.8%
ISTT-R-A	0.0%

Average monthly return for optimal portfolio was 3.58 percent and standard deviation 3.94 percent (one point at the efficient frontier, with risk free rate of 3.95 percent). CROBEX had a lower average monthly return of 0.6 percent and slightly lower standard deviation of 3.25 percent. After that, we calculated the price of a single portfolio unit from the beginning of the period (March 31 2003) until the end of the period (June 31 2004) and we found out that price of our portfolio unit in the beginning of period was 490.5 HRK and in the end was 795.1 HRK, which translates to 62.1 percent increase in 16 months. Meanwhile, CROBEX increased by 8.5 percent.

Table 3. **Prices of constrained optimal portfolio and CROBEX index (historic data)**

Date	Price of the optimal portfolio unit at the end of each month	CROBEX
31.3.2003	490.5	1056.3
30.4.2003	499.2	1120.3
30.5.2003	720.9	1147.3
30.6.2003	650.6	1161.2
31.7.2003	693.3	1137.3
29.8.2003	732.4	1169.1
30.9.2003	642.7	1093.2
31.10.2003	691.8	1141.4
28.11.2003	761.9	1187.6
31.12.2003	784.8	1185.1
30.1.2004	800.0	1197.6
27.2.2004	800.7	1177.3
31.3.2004	747.0	1160.8
30.4.2004	768.6	1175.5
31.5.2004	747.5	1136.5
30.6.2004	795.1	1146.5
Overall return	62.1%	8.5%

After calculating the data for historic period in order to find out which portfolio to 'buy', we moved on to see 'future' performance. Since the time period was so short, we left only two months to track performance of portfolio in the 'future'. We assumed that we bought our portfolio on July 1 2004 and we tracked the performance of portfolio on a daily basis until August 31 2004. Average daily return in this period was 0.39 percent, while CROBEX's average daily return was 0.11 in the same period. Standard deviation was 1.04 and 0.82 percent, respectively. The price of one unit of our optimal portfolio at the beginning of period was 796.2 (July 1 2004) and in the end (August 31 2004) was 846.4, which is a 6.3 percent increase in this two month period or 44.3 annualized. CROBEX increased in the same period by 4.7 percent or 31.7 annualized.

Changes in portfolio prices and CROBEX can be seen on Graph 1.

Graph 2. **Prices of single portfolio unit and CROBEX transformed in base indices**

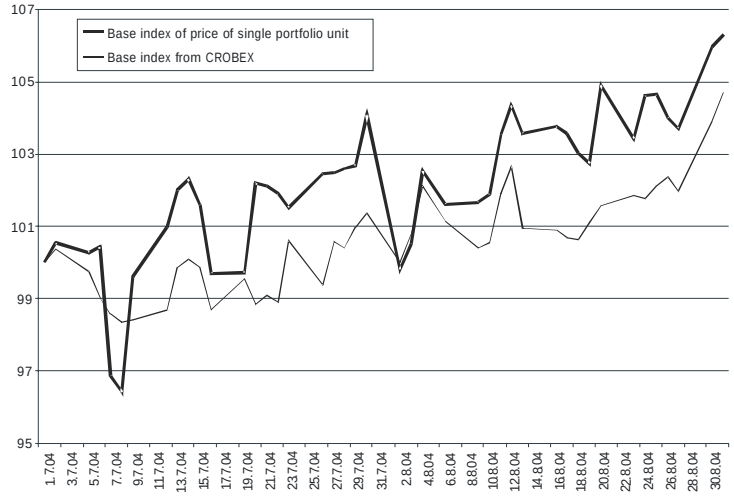


Table 4. **Prices of constrained optimal portfolio and CROBEX index ('future' data)**

Date	Price of single portfolio unit	CROBEX
1.7.04	796.2	1171.6
2.7.04	800.5	1175.8
5.7.04	798.4	1168.8
6.7.04	799.6	1159.9
7.7.04	771.3	1154.9
8.7.04	767.6	1152.2
9.7.04	793.1	1153.0
12.7.04	804.1	1156.3
13.7.04	812.2	1170.1
14.7.04	814.6	1172.6
15.7.04	809.0	1170.0
16.7.04	793.8	1156.2
19.7.04	794.0	1166.2
20.7.04	813.8	1157.8
21.7.04	813.1	1160.9
22.7.04	811.5	1158.5
23.7.04	808.2	1178.5
26.7.04	815.7	1164.3
27.7.04	816.0	1178.5
28.7.04	816.9	1176.4
29.7.04	817.7	1182.9
30.7.04	828.6	1187.6
2.8.04	794.7	1171.7

3.8.04	800.4	1180.8
4.8.04	816.5	1196.4
6.8.04	809.1	1185.0
9.8.04	809.4	1176.3
10.8.04	811.2	1178.0
11.8.04	824.3	1194.1
12.8.04	830.8	1202.6
13.8.04	824.8	1182.8
16.8.04	826.3	1182.1
17.8.04	824.6	1179.5
18.8.04	820.4	1179.1
19.8.04	818.1	1184.7
20.8.04	835.3	1190.0
23.8.04	823.4	1193.3
24.8.04	833.0	1192.4
25.8.04	833.3	1196.3
26.8.04	828.0	1199.4
27.8.04	825.8	1194.8
30.8.04	843.6	1217.4
31.8.04	846.4	1226.7
Overall return	6.3%	4.7%

As can be seen, our optimal portfolio had higher return than market portfolio. But, as return is not the only measure of portfolio performance we used the Sharpe measure. We got 0.37 for optimal portfolio and 0.13 for CROBEX, which shows that our portfolio performed better in this two month period than the market average (the optimal portfolio has higher risk premium return per unit of total risk than CROBEX).

4 Conclusion

The idea behind this work was to use well-known theory and to apply it on data for Croatian stocks.

Since we had only 16 monthly data, for the reasons explained in the text, it is questionable weather this model is applicable to Croatia. In our opinion this is too short of a time series to bring about any valid conclusions. Nonetheless, while we were working on this paper we developed a sound understanding of theory behind the Markowitz's portfolio selection process, and learned how to use it on real data. For the purpose of this work we managed to develop a Mathematica program for unconstrained case, but for the constrained case we had to use Excel's solver function. In the future we will try to find the way to solve constrained optimization problem with Mathematica and compare it with Excel results.

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