# Evaluating Macroeconomic Strategies with a Calibrated Model

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## Abstract

In several catching-up CEE countries we experience an expenditure boom explained by arguments referring to intertemporal consumption optimization. We have calibrated a model assuming externalities from foreign direct investments and a country risk premium, dependent on the debt/GDP ratio. In the model the internal rate of return on marginal savings turned out to be about 18 percent, higher than the level that any estimate of the time preference might justify. Its existence is due to the fact that externalities produced by both saving and investment are not internalized by private agents. Fiscal policy should make the necessary adjustments to approach optimum.

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# 1 Introduction

After a long period of transformational recession CEE countries seem to enter a new phase of development, that of catching-up with the rest of Europe. Having recognized this development, many politicians and many citizens are starting to feel, partly in response to political campaigns, that after a successful political and economic system change time has come for a “welfare system change”. People have suffered enough from the recession prompted by reforms, it is time to consume more now. As most countries have no net external reserves to use – on the contrary, Hungary for example embarked on the transformation with large debts – the source of excess consumption could be future output. This high future output, the prospect of a rapid catching-up process is the main argument that incites people into an exuberant demand for higher wages and higher consumption.

The rationale behind such an intertemporal reallocation of consumption depends on the time preference of consumers. We do not dare put a numerical value to this preference either in the aggregate or for individuals. What we try instead is to calculate the opportunity cost of switching future consumption for present consumption. Or putting it another way, we try to determine the real return of aggregate savings around the current level of net savings in Hungary. Although we had Hungary in mind when calibrating the model, the differences among CEE countries are not so large that our qualitative conclusions would not hold for other countries in the area as well.

The lessons to be learned from the simulations of the model are related to fiscal policy. Our assumption is that foreign capital in Hungary – similarly to other catching-up countries – has a positive external effect on productivity. Similarly, personal or government savings have a positive external effect on the risk premium component of interest rates. Therefore, the decentralized decisions of agents will not lead to an optimum and fiscal policy has the task of correcting the market failure. This correction has an obvious target in a higher stock of resources that produce the externalities. Our model does not provide a recipe for the tools that should be used to correct the market, or to what extent. Basically, the government can influence intertemporal allocation in two ways: by using the tax structure and – in a non-Ricardian world – making savings itself. In our model these tools are not specified. We do not model how the tax system affects consumption[1] or how government savings affect aggregate savings. In addition, we do not consider any other effects of the fiscal policy on output. We know, for example, that fiscal debt has distortionary effects on future working incentives that may lead to future losses of output.[2] We add only one element to the evaluation of the fiscal policy. This element is based on the fact that whatever the channel the fiscal policy uses in correcting market failures, governments have to give account of whether they did their job well. The social opportunity cost that a society faces as a consequence of personal and government choices might be used as a yardstick of their performance.

In Chapter 2 we give a verbal description and justification of the main assumptions, in Chapter 3 we present the equations, in Chapter 4 the simulation results and in Chapter 5 the conclusions.

# 2 Main Assumptions and Features

The model is classical, assuming market clearing. The economy is a small open economy where capital flows are unconstrained but their speed is dampened by adjustment costs. Tradables and non-tradables are distinguished in an implicit way. The model is classical, the price level is not determined, variables are expressed in real terms, the real exchange rate is the price of non-tradables in terms of tradables, normalized with the similar ratio abroad.

The catching-up process is modeled in the following stylized way. Labor supply is constant – more or less in accordance with actual demography. The growth rate of the total factor productivity (TFP) is equal to the world rate in the long run. During the catching-up period the additional growth originates partly from capital accumulation and partly from an additional TFP growth, arising from externalities produced by the inflow of foreign direct capital.

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[1] *Valentinyi (2000) analyzed the effect of the tax system directly on the catching-up process. His model differs from ours as he does not assume externalities and the catching-up process consists of an accumulation of real and human capital.*

[2] *The literature is too rich for referring to it. Blanchard (1990) summarizes the problem in a simple model, Alesina–Ardagna (1998), and Perotti (1999) are some of the recent empirical works on the topic.*

The speed of the catching-up process depends on the speed of the capital accumulation process. Capital stocks (foreign and domestic) reach their steady state anyway, but the speed depends on the country risk premium and the real exchange rate movements. The higher the country risk and the weaker the exchange rate, the faster the capital accumulation.

## 2.1 The Externalities of Foreign Direct Capital

The external effect of FDI is various. It is difficult to separate these effects but we might distinguish two kinds.

1) The agglomeration effect is a well-known concept of regional economics. Economic growth is not smooth geographically, because geographic concentration brings savings (savings in transport, communication, or other transactional or informational costs) in production and sales.[3] This means that each investment improves the profitability of the next entrant, i.e. it produces externality. This phenomenon exists independently from the country of investment origin. Foreign direct investment flows add to this externality only by their net value.

2) In a catching-up country FDI may incorporate a higher level of know-how. This know-how does not remain within a firm but spreads through contacts with other firms: suppliers will be trained or forced to a quality and discipline that comes with a higher production culture of the entrant foreign firm. This external effect – in contrast to the agglomeration effect – depends on the gross FDI inflow.

We do not model the agglomeration effect, the productivity effect of this process is considered in the exogenous TFP term of the production function. We model only the external effect of FDI as a free supplier of know-how.

It is clear that with an increase in the share of foreign capital the know-how supplier effect of foreign capital diminishes. We reflected this feature in our model by specifying the production function in a way that when approaching the steady state, the external effect of FDI approaches 0.

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[3] *See Krugman (1990) and Venables (1996).*

## 2.2 The Saving-Investment Link in an Open Economy

In the introductory economics textbook version of the open economy model, with the opening up of capital markets the period-by-period budget constraint that establishes a link between consumption and investment disappears and the two flows become more independent. We know that capital markets are not perfect and therefore a strong separation of the two systems is not justified.[4] Risks, information constraints and adjustment costs create virtual walls across countries. Capital adjustment is not only gradual, but it does not lead to an equalization of returns either. Country returns contain differing country risk premiums. These premiums depend on the same factor that influences the return in closed capital markets: the level of savings. This brings us back to the feature of the closed capital market economy.

Saving behavior affects the return on capital through two channels: (a) in the medium run a drop in demand weakens the real exchange rate (and decreases the real interest rate) (b) a decrease in the debt level decreases the country risk premium

Let us examine these two factors in more detail.

## 2.3 Demand and the Real Exchange Rate

The idea of the concept of real effective equilibrium exchange rate[5] builds on Dornbusch (1980) who says that an increase in demand makes relative costs of non-tradables higher because tradables demand is met by imports, while output has to switch to non-tradables in order to meet increased demand. This means that the real exchange rate will appreciate. Because of the real interest rate parity this means a higher interest rate. This way an increase in consumption crowds out investment like in the closed capital market economy.

This feature is included into our model but it works only in the medium run. In the long run, the real exchange rate is independent from demand. In the long run, relative costs in the non-tradable sector will not increase because there are no sector-specific production factors by assumption, allowing for factor flows to equalize relative returns across

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*[4] See Feldstein–Horioka (1980) for the first demonstration of this "puzzle".*

*[5] See for example Faruqee (1995), Stein (1999).*

sectors at the starting rates. This adjustment process leads to the purchasing power parity (PPP) or the Balassa-Samuelson path as the more general case.[6] The half-life of the adjustment process is about 4-5 years according to the "international consensus" view.[7]

This way our model combines the PPP and the "sustainability" approach to the equilibrium real exchange rate. The difference between the two approaches lies in the assumption about the speed of adjustment of production factors. The sustainability approach considers the convergence to PPP to be too slow to be taken into account at all. In our approach the half-life of 4-5 years is too short to be disregarded. As a consequence, the real exchange rate and the trade balance are related only in the medium run, while in the long run PPP (Balassa-Samuelson equilibrium) holds.

Stein (1999) defines a concept of the long-term equilibrium real exchange rate in his NATREX[8] model. In this definition, in the long run, capital stock is in a steady equilibrium state. This definition would be useful for conclusions about the real exchange rate only if the adjustment process of capital stocks were faster than the adjustment process of production factors across tradable and non-tradable sectors. Otherwise, PPP is the long-term equilibrium. Economic history shows that the processes leading to debt accumulation are very slow, they may take decades.[9] The process towards PPP may be slow, but in any case faster than this.

We calibrated the model to take into account these differences in the speed of adjustment. This way a fiscal shock appreciates the real exchange rate in the short run, in the long run however – in contrast to the NATREX model – the rate is determined not by a drop in demand because of the debt burden, but by the convergence process to the Balassa-Samuelson path. In the long run, there is no direct interaction between net exports and the real exchange rate. If demand depreciated the real exchange rate, this

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[6] *According to the Balassa-Samuelson hypothesis, productivity in the tradable sector increases faster than that in the non-tradable sector and the gap between the productivity rates depends on the rate of aggregate productivity growth. Therefore the real exchange rate of fast growing economies strengthens.*

[7] *See Rogoff (1996) about the international consensus.*

[8] *NATREX (Natural rate of equilibrium exchange rate) and similar concepts are based on the classical assumption of zero output gap (called internal equilibrium), and the equilibrium real exchange rate is defined as the common slope of the production possibilities frontier and the utility function. While these models consider the strictly convex production possibilities frontier as given even in the long run, in our model the elasticity of substitution between sectoral outputs becomes infinite in the long run.*

[9] *A formal model for this behavior is given in Simon-Várpalotai (2001).*

could happen only because the accumulated debt leads to a higher country risk interest premium. This in turn suppresses investments and the suppressed foreign component of investments affects TFP and the real exchange rate through the Balassa-Samuelson effect.

## 2.4 The Country Risk Premium

As discussed before, the existence of country risk constrains the wedge between consumption and investment in a country, partly taking over the role of an instantaneous budget constraint. This allows the interest rate and capital returns to differ from international rates permanently, preserving the feature of the closed economy Ramsey-model, where impatient consumer behavior leads to a lower steady state capital stock and output.[10]

The country risk has to be an important issue in macroeconomic policy as individual risk taking renders negative externalities.[11] Each borrower adds to the country risk premium, but the cost of additional risk caused to others is not internalized in his/her own calculations. Macroeconomic policy has the task of correcting this market failure.

Before discussing the calibration of the impact of risk, let us make some remarks about the nature of the risk premium before and after joining the currency union. Indebtedness leverages an economy and thus magnifies the relative variance of its income. For foreign investors who sell in the country this means a higher demand risk and accordingly a higher revenue risk. For lenders of financial assets this means a higher default risk and exchange rate risk.

Upon joining the currency union, the currency risk disappears but the leverage of the economy does not change. The total risk depends on whether the flexible exchange rate system itself added to the aggregate risk or not, and whether it was a stabilizing or a destabilizing factor on the economy. We do not take up this issue of discussion. In the model we take a middle road by assuming that the total risk of investments does not change by this system change. Technically, we do not distinguish between financial and

[10] *In our model the risk of higher consumption today is taken into account only as a country risk premium, which means practically a default risk. The risk for the consumer should have been specified in the consumption function, if we had used a structural formulation.*

[11] *See Harberger (1986).*

direct investment risk, rather all the risks are considered simply as a source of interest rate premium. In case of modeling the effect of the currency union, we had to give account of the distribution of risk between financial and physical capital.

# 3 Model Equations

As our model describes an infinitely growing economy, we normalize growing variables by output. This way we analyze the path towards a steady state of the rates of variables to output. Normalized variables are given in small letters, while steady state values are denoted by a bar above the variable.

## 3.1 Output

Output is determined by a Cobb-Douglas production function homogenous at first degree:

$$Y_t = A_t K_{f,t-1}^{\alpha_f} K_{d,t-1}^{\alpha-\alpha_f} L^{1-\alpha},$$

where $L$ labor supply is fixed ( $L = 1$ ), $K_f$ and $K_d$ are foreign and domestically owned capital, $A_t$ productivity, including the external effect of foreign capital ratio:

$$A_t = A_0 (1+\mu^{tfp})^t e^{\gamma \frac{K_{f,t-1}}{Y_{t-1}}},$$

where $A_0$ is a scaling factor of output, $:_{tfp}$ is the exogenous constant component of TFP growth, and $e^{\gamma \frac{K_{f,t-1}}{Y_{t-1}}}$ is the output generating externality, implied by the ratio of foreign owned capital.[12] For the sake of easier calculation we transformed capital stock variables to rates:

(D1) $$Y_t = A_0 (1+\mu_{tfp})^t e^{\gamma k_{f,t-1}} k_{f,t-1}^{\alpha_f} k_{d,t-1}^{\alpha-\alpha_f} Y_{t-1}^{\alpha} L^{1-\alpha}.$$

Growth rate of output:

(D2) $$g_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}}.$$

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[12] *We can easily check whether the production function is homogenous at first degree by multiplying the explanatory variables by a constant.*

The change in $Y_t / Y_t^W$ , the output level relative to the world, depends on the difference between $g_t$ domestic and $g_W$ (assumed constant) world growth rates :

(D3) $$Y_t / Y_t^w = \frac{1+g_t}{1+g^w} Y_{t-1} / Y_{t-1}^w .$$

## 3.2 Capital Accumulation and Investment

Equations for capital accumulation are standard, where * is the depreciation rate:[13]

(D4) $$k_{f,t} = \frac{1-\delta}{1+g_t} k_{f,t-1} + i_{f,t}$$

(D5) $$k_{d,t} = \frac{1-\delta}{1+g_t} k_{d,t-1} + i_{d,t}$$

The dynamics of $i_{f,z}$ and $i_{d,t}$ investments are described in the spirit of Tobin-q theory similarly to a model with quadratic adjustment costs $\left(\Psi(i,k) = i + \beta_k \frac{i^2}{2k}\right)$ .
Accordingly, investments have a positive correlation to profits over alternative returns cumulated into the future ( $A_{f,t}$ and $A_{d,t}$ )

(D6) $$i_{d,t} = \frac{\Pi_{d,t}}{\beta_{kd}} + \frac{\delta + \bar{g}}{1+\bar{g}} k_{d,t-1}$$

(D7) $$i_{f,t} = \frac{\Pi_{f,t}}{\beta_{kf}} + \frac{\delta + \bar{g}}{1+\bar{g}} k_{f,t-1},$$

where $\frac{\delta+\bar{g}}{1+\bar{g}} k_{d,t-1}$ and $\frac{\delta+\bar{g}}{1+\bar{g}} k_{f,t-1}$ are parts of investment that maintain an unchanged capital-output ratio at a $\bar{g}$ growth rate.

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[13] *Written in level form:* $K_{f,t} = (1-\delta) K_{f,t-1} + I_{f,t}$ . *Dividing by* $Y_t$*:*
$\frac{K_{f,t}}{Y_t} = (1-\delta)\frac{K_{f,t-1}}{Y_{t-t}}\frac{Y_{t-1}}{Y_t} + \frac{I_{f,t}}{Y_t}$ *, and using the definition of* $g_t$ *and regrouping produces the above formula.*

## 3.3 Consumption

We use a reduced form equation for describing consumption behavior. $c^t$ consumption is a linear function of $y_t^{lab}$ labor income and $w_t$ net wealth. This functional form is standard in econometric models. Its advantage is that it is easy to estimate and it leads to a finite and positive steady state consumption-output ratio. Calibration models are often based on the Euler equation that assumes a representative consumer living infinitely. The advantage of the latter is that in this case the model is built on the structural parameters of behavior of the representative consumer (Chatterjee, Sakoulis and Turnovsky, 2003) use this approach when simulating the effect of capital flows on growth in open economies). We discarded this approach because recent results in theory show that the assumption of a homogenous consumer and infinite horizon simply does not give a good description of aggregate consumption (see Carroll, 2001). The behavior of the rich differs from the behavior of the poor so much that taking averages is not a fruitful approach. Although the interest rate does affect consumption even in the new models of consumption, the impact is so small that we took the liberty of disregarding it for the sake simplicity.

(D8) $$c_t = \beta_w \frac{w_{t-1}}{1+g_t} + \beta_y y_t^{lab}.$$

In the above equation the coefficients of $\exists_w$ and $\exists_y$ have been calibrated in a way that the consumption ratio converges to its $\overline{w}$ steady state value with a half-life of $0$[14]:

(P1) $$\beta_w = 1 + \overline{r}^{pf} - (1+\overline{g})\exp\left(\frac{\ln 0.5}{\eta}\right)$$

(P2) $$\beta_y = 1 - \left[1 - \exp\left(\frac{\ln 0.5}{\eta}\right)\right]\frac{\overline{w}}{\overline{y}^{lab}}.$$

[14] *See Appendix A. for a detailed derivation.*

## 3.4 Incomes, Asset balances, Returns

$B_{f,t}$ and $B_{d,t}$ the return on capital is determined by the Cobb-Douglas consumption function by profit maximizing behavior. Production factors get a fixed share from income:[15]

(D9) $$\pi_{f,t} = (1+g_t)\frac{\alpha_f}{k_{f,t-1}}$$

(D10) $$\pi_{d,t} = (1+g_t)\frac{\alpha - \alpha_f}{k_{d,t-1}}.$$

The $y_t^{lab}$ labor income is the rest of income after deducting capital income:

(D11) $$y_t^{lab} = 1 - \frac{\pi_{f,t}k_{f,t-1} + \pi_{d,t}k_{d,t-1}}{1+g_t}.$$

$r_t^d$, domestic real interest rate is determined by the interest rate parity corrected for the risk premium:

(D12) $$r_t^d = \theta(q_t - q_{t+1}) + \rho_t + r^w,$$

where $r^w$ is the constant "world interest rate", $\Delta_t$ the country risk premium, $q_t$ the real exchange rate (price of non-tradables in terms of tradables), *2* a constant weight parameter that converts a change in the relative price of the two sectors into a change in the relative price of non-tradables to the aggregate of both sectors. This makes the right-hand side consistent with the definition of the real interest rate.[16]

The $\Delta_t$ country-risk premium depends on net financial assets ($nfa_t$):[17]

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[15] *The profit maximization condition:* $\pi_{f,t} = \frac{\partial Y_t}{\partial K_{f,t-1}}$ *. The formula above was produced by using the definition of* $g_t$.

[16] *In equation (D12) we approximated relative rates of changes with differences in the rates.*

[17] *Debt as a risk factor may be defined in several ways. The traditional measurement is the ratio of net foreign financial assets (interest-bearing debt) to income. The alternative concept adds net real assets to the numerator. The choice depends on whether we perceive real assets to be a good hedge against labor income risk. In the model we followed the usual approach that considers only interest bearing debt as a risk factor.*

(D13) $$\rho_t = \max\{0; -\beta_\rho nfa_t\}.$$

The definition of variables $A_{f,t}$ and $A_{d,t}$ used in equations (D6) – (D7):

(D14) $$\Pi_{f,t} = \sum_{k=t+1}^{\infty} \left[ \left( \pi_{f,k} - \delta \right) - r_k^d \right]$$

(D15) $$\Pi_{d,t} = \sum_{k=t+1}^{\infty} \left[ \left( \pi_{d,k} - \delta \right) - r_k^d \right]$$

When substituting from equation (D12), we see that an investment decision depends on the foreign interest rate, risk premium, and the real exchange rate as cost factors. The latter in a way that expected appreciation increases expected return.

Net foreign financial assets:[18]

(D16) $$nfa_t = \frac{(1+r_t^d) nfa_{t-1} - \pi_{f,t} k_{f,t-1}}{1+g_t} + tr_t + i_{f,t},$$

where $tr_t$ is the trade balance, and $i_{f,t} - B_{f,t}\, k_{f,t-1} /(1+g_t)$ the net foreign capital related flows (FDI minus profit flows).

For the sake of simplicity we assume that local residents do not directly invest abroad. A net wealth of the country ($w_t$) is then:

(D17) $$w_t = nfa_t + k_{d,t}.$$

The return on net wealth is the sum of the return on net financial and net real assets:

(D18) $$r_t^{pf} = \frac{(\pi_{d,t} - \delta) k_{d,t-1} + r_t^d nfa_{t-1}}{k_{d,t-1} + nfa_{t-1}}.$$

Identity of the trade balance:

(D19) $$tr_t = 1 - c_t - i_{f,t} - i_{d,t}.$$

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[18] *Written in a level form:* $NFA_t = (1+r_t^d) NFA_{t-1} + TR_t + I_{f,t} - \pi_{f,t} K_{f,t-1}$. *Similarly as in the equation for capital accumulation, dividing through by* $Y_t$ *we arrive at:* $\frac{NFA_t}{Y_t} = (1+r_t^d) \frac{NFA_{t-1}}{Y_{t-1}} \frac{Y_{t-1}}{Y_t} + \frac{TR_t}{Y_T} + \frac{I_{f,t}}{Y_t} - \pi_{f,t} \frac{K_{f,t}}{Y_{t-1}} \frac{Y_{t-1}}{Y_t}$ *, and using the definition of* $g_t$ *we get the formula above.*

## 3.5 The Real Exchange Rate

We assume that during the catching-up process the real exchange rate appreciates because of the Balassa-Samuelson effect. The rate of this appreciation depends on the growth difference to the world rate:

(D20)
$$q_{BS,t} = q_{BS,t+1} - \beta_{BS}\left(g_t - g^w\right),$$

where higher $q_{BS,t}$ means appreciation and $lim_{t\to\infty}\, q_{BS,t} = 1$ . This constraint makes the domestic relative price equal to the world rate in a steady state. The defined $q_{BS}$ path is an equilibrium path that actual rate converges to along the path described below.

The distribution of consumer and investment demand between tradables and non-tradables is determined by maximizing a CES function:

(D21)
$$\begin{aligned} &\max\left[A_{tr}\left(c_{tr,t} + i_{tr,t}\right)^{-\beta_d} + A_{ntr}\left(c_{ntr,t} + i_{ntr,t}\right)^{-\beta_d}\right]^{-\frac{1}{\beta_d}} \\ &\text{s.t. } \left(c_{tr,t} + i_{tr,t}\right) + q_t\left(c_{ntr,t} + i_{ntr,t}\right) = c_t + i_{f,t} + i_{d,t} \end{aligned}$$

where *tr* and *ntr* subscripts denote the tradable and non-tradable sectors, $\exists_d$ , $A_{tr}$ , and $A_{ntr}$ are parameters of the CES function. From the first order conditions:

(D22)
$$q_t = \frac{A_{ntr}}{A_{tr}}\left(\frac{c_{ntr,t} + i_{ntr,t}}{c_{tr,t} + i_{tr,t}}\right)^{-\beta_d - 1}.$$

Let us assume that consumer and investor CES functions are the same:

(D23)
$$\frac{c_{tr,t}}{c_{ntr,t}} = \frac{c_{tr,t} + i_{tr,t}}{c_{ntr,t} + i_{ntr,t}}$$

(D24)
$$\frac{i_{tr,t}}{i_{ntr,t}} = \frac{c_{tr,t} + i_{tr,t}}{c_{ntr,t} + i_{ntr,t}}.$$

The composition of the total output in terms of tradables,

(D25)
$$y_t = y_{tr,t} + q_t y_{ntr,t}$$

is determined by a CET (constant elasticity of transformation) production function:

$\left[B_{tr,t}y_{tr,t}^{-\beta_s}+B_{ntr,t}y_{ntr,t}^{-\beta_s}\right]^{-\frac{1}{\beta_s}}$, where $B_{tr,t}$ , $B_{ntr,t}$ are parameters changing in time and $\exists_s$ is a constant parameter. The marginal rate of transformation is equal to the real rate of exchange:

(D26) $$q_t = b_t\left(\frac{y_{ntr,t}}{y_{tr,t}}\right)^{-\beta_s-1},$$

where $b_t = \frac{B_{ntr,t}}{B_{tr,t}}$ .

The condition that the non-tradable market is closed:

(D27) $$y_{ntr,t} = c_{ntr,t} + i_{ntr,t}.$$

In the CET function the $\frac{B_{ntr,t}}{B_{tr,t}}$ ratio is not constant. The reallocation of production factors from one sector into another may be interpreted as a change in the weighting parameters. There is a "Balassa-Samuelson weighting parameter" path, which equates the actual $q_t$ to the $q_{BS,t}$ real exchange rate path:[19]

(D28) $$b_t^{BS} = q_{BS,t}\left(\frac{1-tr_t}{\left(\frac{A_{tr}}{A_{ntr}}q_{Bs,t}\right)^{\frac{1}{\beta_d+1}}+tr_t q_{BS,t}}\right)^{\beta_s+1}.$$

Various shocks result in the differences in sectoral wages and profits, but the adjustment process of factor allocation in the weights of supply change and the real exchange rate converges to the Balassa-Samuelson path. We use a reduced formulation without explicitly modeling the adjustment process:

(D29) $$b_t = b_t^{BS} - \lambda_s\left(b_{t-1}^{BS} - b_{t-1}\right).$$

## 3.6 Calibration of the Parameters

The model has 29 variables, $Y_t$ , $Y_t/Y_t^w$ , $g_t$ , $k_{d,t}$ , $k_{f,t}$ , $i_{d,t}$ , $i_{f,t}$ , $c_t$ , $B_{d,t}$ , $B_{f,t}$ , $y_t^{lab}$ , $r_t^d$ , $\Delta_t$ , $A_{d,t}$ , $A_{f,t}$ , $nfa_t$ , $w_t$ , $r_t^{pf}$ , $tr_t$ , $q_{BS,t}$ , $q_t$ , $c_{ntr,t}$ , $i_{ntr,t}$ , $c_{tr,t}$ , $i_{tr,t}$ , $y_{tr,t}$ , $y_{ntr,t}$ , $b_t^{BS}$ , $b_t$ in 29 equations. There are two additional equations that determine the values of $\exists_w$ and $\exists_y$.

[19] *See Appendix B for the derivation.*

Parameters of the model: $A_0$, $\forall$, $\forall_f$, $:_{tfp}$, $($, $(_{kf}$, $(_{kd}$, $\exists_{kf}$, $\exists_{kd}$, $*$, $2$, $r^W$, $g^W$, $\exists_\Delta$, $\exists_{BS}$, $\frac{A_{ntr}}{A_{tr}}$, $\exists_d$, $\exists_s$, $8_s$, $0$, $\overline{w}$.

For the starting value of capital ratios we used the estimates of Darvas and Simon (1999) and Pula (2003), the share of foreign capital was estimated from accumulating FDI data: $k_d = 1.1$ , $k_f = 0.4$ . The rest of stock values for the beginning of 2003 were taken from the national income accounts: $nfa$ = -0.25 and $w$ = 0.85 comes from an identity. For the $Y_t / Y_t^W$ relative level of income we assumed $Y / Y^W$ = 50%, which is the Hungarian level relative to the average of the European Union.

The $r^W$ world interest rate was taken to be 4%, the depreciation rate 9%. The $2$ relative weight of tradables was assumed as 0.5. The coefficient of the country risk, measured as the rate of debt, is $\exists_\Delta$ = 0.1, meaning that a 10 percent increase in debt raises the interest rate by 1 percentage point.[20] The $\overline{w}$ steady state wealth ratio was chosen by the assumption that $\overline{nfa} = 0$. The $0$ half-life parameter is 10 years.

The production function. $A_0$ = 0.932438 was chosen so that in the first year of simulation output equals 1. Capital share is the international usually observed $\forall$ = 0.3. Total factor productivity growth rate was calibrated in a way that in a steady state growth equals the world rate, $g_W$ = 2% , therefore $\mu = \left(1 + \overline{g}\right)^{1-\alpha} - 1 = 0.01396$ .

The external effect of foreign capital was calibrated in a way that actual GDP growth should fit the values calculated from the calibrated model in the period of 1996-2003. This criterion gave a coefficient of the external effect $($ = 0.4, meaning that a 1 percent increase in the foreign capital ratio increases TFP by 0.4 percentage point. This figure is higher than the 0.21 estimated by Jakab and Kovács (2002) on the Central East European panel data. Barrell and Pain (1997) made an estimation for Great Britain. In Great Britain the role of foreign capital as a supplier of know-how externality might be small, and instead the estimated coefficient reflects the agglomeration effect (Krugman, 1991; Venables, 1996) – as it is assumed by the authors as well. Thus their coefficient of 0.27 is not comparable to our figure, especially because in our model the agglomeration effect is included in the exogenous component, while they do not specify an exogenous TFP at all.

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[20] *Edwards (1984) arrived at a half-elasticity of 0.6-1.0 in a panel estimation. At a level of about 2% this is roughly the corresponding figure to our linear coefficient of 0.1. Our figure is definitely more cautious than the assumed 0.4 in Fagan, Gaspar and Pereira (2002). The relation between risk and debt is presumably non-linear, but in our simulations we do not depart so far from the base scenario to take this into account.*

Figure 1. **Contribution to GDP growth of production factors, exogenous TFP and externalities (1996. = 100 %) according to (D1) production function with calibrated parameters**

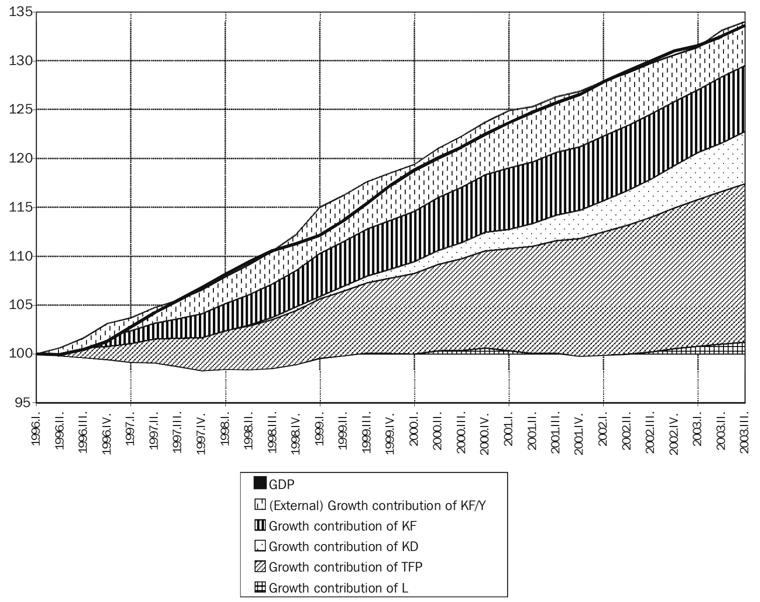



Figure 1 shows how various components contributed to GDP growth.

We assumed that in the steady state Hungarian productivity is at the 80 percent level of the European average. This is in line with the arguments of Darvas and Simon (1999).

Investment behavior is explained in line with the Tobin-q theory. In order to estimate the speed of adjustment, very much empirical work can be found in the literature. The estimated parameters are in a broad range. In the model $\exists_{k_f}$ and $\exists_{k_d}$ is the parameter determined by adjustment costs. Estimates in the empirical literature on the adjustment cost parameter range between 1.4 and 16.1.[21] We chose a value between the two extremes, assuming $\exists_{k_f} = \exists_{k_d} = 10$.

[21] *Summers (1981) estimated a value of 16.1, Eberly (1997) arrived at a range of between 1.4 and 3 using micro data. Cummins, Hasset and Oliner (1997) using US company data estimated parameters of between 5 and 10.*

In the real exchange rate block the Balassa-Samuelson appreciation effect of additional growth, $\exists_{BS} = 0.5$, $\frac{A_{ntr}}{A_{tr}} = 1$. We chose the exchange rate elasticities of demand and supply, $\exists_d = 2$ , $\exists_s = -3$ to make a 1 percentage point deterioration of the trade balance consistent with a 2 percent appreciation of the real exchange rate. This number is in line with the estimations of Jakab and Kovács (2002) for exports and imports. At the starting date the real exchange rate is in equilibrium by assumption. This assumption is indifferent for the policy lessons derived from the model and it has no relevance to the actual real exchange rate either, as the real exchange rate in our model is the rate of the hypothetical case when the output gap equals 0. In the short run, for example during a disinflation process, this assumption is probably untenable.

In the supply function the $8_s$ parameter that determines the speed of adjustment across sectors was chosen to be consistent with the 5 years half-life of the convergence to the Balassa-Samuelson equilibrium, $8_s = 0.87055$.

In Table 1 we summarized the assumptions on parameters, starting values, and the implied steady state values.

Table 1. **Parameters, starting and steady state values**

| | | | | | |
|---|---|---|---|---|---|
| **Parameters** | | | **Calculated coefficients** | | |
| $0$ | = | 10 | $\exists y$ | = | 0.70981 |
| $\overline{w}$ | = | 1.56923 | $\exists w$ | = | 0.16204 |
| $\forall$ | = | 0.3 | | | |
| $\forall_f$ | = | 0.1 | | | |
| ( | = | 0.4 | | | |
| $:_{tfp}$ | = | 0.01396 | | | |
| $\exists_{kf}$ | = | 10 | | | |
| $\exists_{kd}$ | = | 10 | | | |
| * | = | 0.09 | | | |
| $2$ | = | 0.5 | **Steady state values** | | |
| $r^w$ | = | 0.04 | $\overline{r}^d$ | = | 0.05 |
| $g^w$ | = | 0.02 | $\overline{g}$ | = | 0.02 |
| $\exists_\Delta$ | = | 0.1 | $\overline{\Delta}$ | = | 0 |
| $\exists_{BS}$ | = | 0.5 | $\overline{c}$ | = | 0.74615 |
| $\frac{A_{ntr}}{A_{tr}}$ | = | 1 | $\overline{i}_f + \overline{i}_d$ | = | 0.23077 |
| $\exists_d$ | = | 2 | $\overline{tr}$ | = | 0.02308 |
| $\exists_s$ | = | -3 | $\overline{y}^{lab}$ | = | 0.7 |
| $8_s$ | = | 0.5 | $\overline{B}$ | = | 0.13 |
| | | | $\overline{r}^{pf}$ | = | 0.05 |
| ***Starting values*** | | | | | |
| $k_d$ | = | 1.1 | $\overline{k}_d$ | = | 1.56923 |
| $k_f$ | = | 0.4 | $\overline{k}_f$ | = | 0.78461 |
| $nfa$ | = | -0.25 | $\overline{nfa}$ | = | 0 |
| $Y/Y^w$ | = | 0.5 | $\overline{Y/Y^w}$ | = | 0.77172 |

# 4 Simulations

## 4.1 Catching-up Paths

Firstly, we show growth (catching-up) paths implied by the parameters of Table 1. We calculate 4 alternative scenarios. In the case that we consider the most probable, both the externality of foreign capital and the risk premium are important factors. In the other variants we show what will happen if one of the two factors or both are omitted. By showing the alternatives, we try to give a picture of the sensitivity of the results on critical assumptions of the model.

In the first column we show the paths calculated on the assumption that foreign capital has an external effect. The two paths differ on the assumption whether indebtedness increased the risk premium. In the absence of a risk premium the catching-up process is faster.

In the second column we calculated with a model where productivity growth is entirely exogenous. This exogenous productivity growth is equal to the sum of exogenous and externalities-driven growth, calculated in the baseline case where both externalities and risk premium exist. Here again the existence of risk constrains the speed of catching-up. However, as in this case the role of foreign capital is minor, the existence of the risk premium does not make much difference.

Figure 2. **Catching-up paths in four variants**

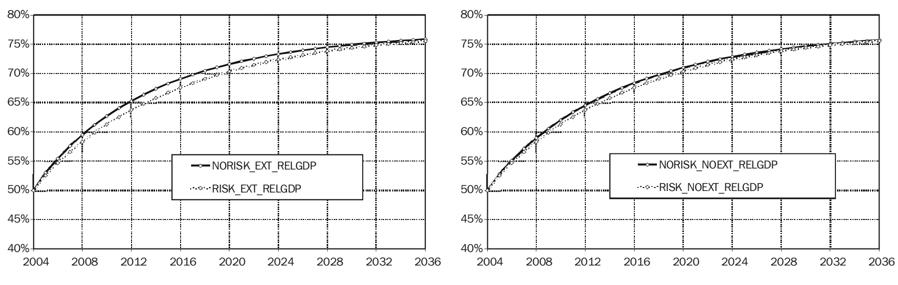
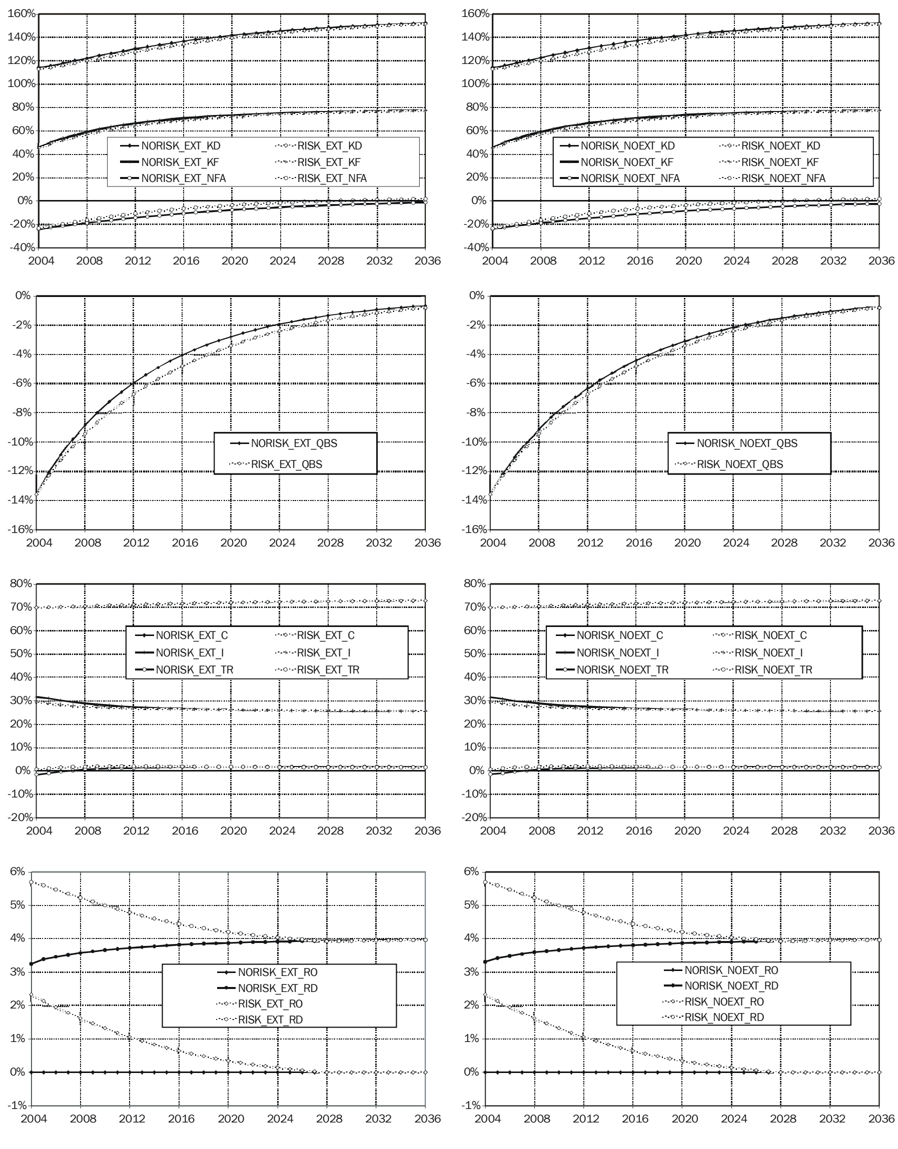


Figure 2. **continued**




*Note: In the right-hand column the assumption holds that foreign capital has an external effect, while this assumption is dropped from the left-hand column. Black curves indicate variants where endogenous risk premium is absent, while gray curves show variants with an endogenous risk premium.*

## 4.2 Effects of Consumption Shocks

None of the four paths can be interpreted as an optimum, in the sense that it maximizes the value of consumer utility functions. We discarded this approach when bringing the consumption function from outside into the model. We could have chosen to calculate the intertemporal optimum with a representative consumer or a social preference function in mind, then calculate a "suboptimum" as the solution of the decentralized task with the distortions created by externalities and calculate the optimal tax system that leads to the social optimum. Although we do not deny that the theoretical rigor of such an approach is a merit, we believe that the parameters of a social preference function are rather uncertain so it is better to keep the model used for drawing policy lessons simple because its results are easier to interpret. Therefore, we simply made a present value calculation that answers the question of what the intertemporal rate of substitution at the baseline path between consumption is today and tomorrow.

The consumption shocks need not be interpreted as fiscal consumption shocks. However, if we want to draw policy conclusions, it is natural to consider them as consumption shocks forced out by the fiscal policy. It is reasonable to assume this possibility because Ricardian equivalence exists only in the infinite horizon representative consumer case.
We calculated the effect of two kinds of shocks:

1) In a transitory consumption shock the consumption ratio increases by 1 percentage point in the first period (year) but the steady state wealth ratio does not change.

2) In a permanent consumption shock the consumption ratio increases by 1 percentage point in the first period (year) and the steady state wealth ratio decreases by the same 1 percentage point.

The consumption shock exerts its influence through the following channels. A transitory shock increases demand that appreciates the currency and – because of the interest rate parity – it raises the interest rate. At the same time, interest rates are affected by the resulting increase in the risk premium. A higher interest rate decreases expected net profits and investments including FDI will be lower, reducing productivity growth. Later consumption adjusts to meet the targeted wealth ratio and investments rebound to put the economy onto the original steady state level. In case of a permanent shock the targeted wealth ratio is lower, which means a permanently higher risk premium and lower output and productivity level in the steady state.

A summary indicator of the shock impact is the internal interest rate. When used as a discount factor, the present value of future foregone consumption equals the gain in present consumption, or putting it another way, the sum of differences to the baseline in consumption flows equals 0. As Table 2 shows, if we do not believe in externalities and risk premiums, we arrive at a 4.2%–5.0% interest rate. This is significantly higher than the world real interest rate adjusted with the Balassa-Samuelson effect,[22] but not so high that it could not easily be explained by a high time preference. However, if we assume the existence of a risk premium, the implicit interest rate increases to more than 7.4 percent, and the inclusion of externalities brings the interest rate to the level of 17.2-18.1 percent. This is a shockingly high rate, higher than anything observed on the markets normally.

Table 2. **Implicit interest cost (return of aggregate saving) at the present level of indebtedness**

| | With foreign capital externalities | | Without foreign capital externalities | |
|---|---|---|---|---|
| | **With risk premium** | **Without risk premium** | **With risk premium** | **Without risk premium** |
| Transitory shock | 17.2% | 6.3% | 10.0% | 5.0% |
| Permanent shock | 18.1% | 5.1% | 7.4% | 4.2% |

We tested the robustness of the results in a sensitivity analysis. We changed the parameter values by 50 percent one by one and calculated the results again. The differences from the baseline are shown in Table 3.

As we see for most of the parameters, even a 50 percent change does not result in a change in the internal interest rate higher than 1 percent. In the variants without country risk premium the sensitivity is even lower. The parameters that have the strongest impact on the interest rate are the parameters influencing capital accumulation and productivity directly, such as $($, the external effect of foreign capital, $\exists_{kd}$ and $\exists_{kf}$ , the adjustment costs of investment, and $\exists_{\Delta}$ , the risk premium coefficient. Even these parameters cause changes in the range of 2 percent only and the highest value is 3.2 percent.

Summing up the results from the sensitivity analysis, we can say that the figures in Table 2 are robust, the conclusions do not change qualitatively even if some parameters change considerably.

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[22] *In the presence of the Balassa-Samuelson effect real interest rate parity holds only in terms of the tradable sector, while the basket will appreciate.*

Table 3. **Analysis of parameter sensitivity: Implicit interest costs of a unit change in consumption, difference from the baseline**

| | | Transitory shock | | | | Permanenr shock | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | With foreign capital externalities | | Without foreign capital externalities | | With foreign capital externalities | | Without foreign capital externalities | |
| | | With risk premium | Without risk premium | With risk premium | Without risk premium | With risk premium | Without risk premium | With risk premium | Without risk premium |
| $\eta=10$ | $\eta=5$ | -0.7% | 1.1% | 0.2% | 1.4% | -0.7% | -0.1% | -0.6% | 0.0% |
| | $\eta=15$ | 0.4% | -0.5% | 0.1% | -0.5% | 0.3% | 0.0% | 0.4% | 0.0% |
| $\gamma=0.4$ | $\gamma=0.3$ | -1.6% | -0.5% | 0.1% | 0.1% | -2.5% | -0.2% | 0.0% | 0.1% |
| | $\gamma=0.5$ | 1.7% | 0.6% | 0.0% | -0.1% | 2.6% | 0.3% | 0.0% | -0.1% |
| $\beta_{kf}=\beta_{kf}=10$ | $\beta_{kf}=\beta_{kf}=5$ | 2.1% | 1.3% | 0.8% | 0.1% | 1.3% | 0.5% | 0.5% | 0.0% |
| | $\beta_{kf}=\beta_{kf}=15$ | -1.4% | -0.5% | -0.6% | 0.0% | -0.9% | -0.2% | -0.4% | 0.0% |
| $\rho=0.09$ | $\rho=0.08$ | 0.2% | 0.1% | -0.1% | 0.0% | 0.6% | 0.0% | -0.1% | 0.0% |
| | $\rho=0.10$ | -0.2% | -0.1% | 0.1% | 0.0% | -0.5% | 0.0% | 0.1% | 0.0% |
| $r_w=0.04$ | $r_w=0.03$ | -0.5% | -0.9% | -1.1% | -1.1% | -0.1% | -1.6% | -1.2% | -1.5% |
| | $r_w=0.05$ | 0.6% | 0.9% | 1.1% | 1.1% | 0.2% | 1.3% | 1.2% | 1.2% |
| $\beta_\rho=0.1$ | $\beta_\rho=0.05$ | -0.3% | 0.0% | -1.6% | 0.0% | -3.2% | 0.0% | -1.1% | 0.0% |
| | $\beta_\rho=0.15$ | 2.2% | 0.0% | 1.4% | 0.0% | 2.0% | 0.0% | 0.8% | 0.0% |
| $g=0.02$ | $g=0.01$ | -0.1% | 0.1% | 0.1% | 0.2% | -0.1% | 0.4% | 0.2% | 0.3% |
| | $g=0.03$ | 0.1% | -0.1% | -0.1% | -0.2% | 0.1% | -0.7% | -0.2% | -0.5% |

**Table 3. continued**

| | | Transitory shock | | | | Permanenr shock | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | With foreign capital externalities | | Without foreign capital externalities | | With foreign capital externalities | | Without foreign capital externalities | |
| | | With risk premium | Without risk premium | With risk premium | Without risk premium | With risk premium | Without risk premium | With risk premium | Without risk premium |
| $\beta_{bs}=0.5$ | $\beta_{bs}=0.25$ | 0.3% | 0.4% | 0.3% | 0.3% | 0.0% | 0.2% | 0.1% | 0.1% |
| | $\beta_{bs}=0.75$ | -0.3% | -0.5% | -0.3% | -0.3% | 0.0% | -0.2% | -0.1% | -0.1% |
| $A_{ntr}/A_{tr}=1$ | $A_{ntr}/A_{tr}=0.5$ | -0.2% | -0.3% | -0.2% | -0.2% | -0.1% | -0.1% | 0.0% | -0.1% |
| | $A_{ntr}/A_{tr}=1.5$ | 0.2% | 0.2% | 0.1% | 0.1% | 0.1% | 0.1% | 0.0% | 0.0% |
| $\beta_d=2$ | $\beta_d=1$ | -0.4% | -0.5% | -0.3% | -0.3% | -0.2% | -0.2% | -0.1% | -0.1% |
| | $\beta_d=3$ | 0.3% | 0.3% | 0.2% | 0.2% | -0.1% | -0.1% | 0.0% | -0.1% |
| $\beta_s=-3$ | $\beta_s=-4$ | 0.6% | 0.8% | 0.5% | 0.5% | 0.3% | 0.4% | 0.1% | 0.1% |
| | $\beta_s=-2$ | -1.1% | -1.4% | -0.9% | -0.8% | -0.4% | -0.5% | -0.2% | -0.2% |
| $\theta=0.5$ | $\theta=0.25$ | -1.4% | -1.7% | -1.1% | -1.0% | -0.7% | -0.6% | -0.2% | -0.2% |
| | $\theta=0.75$ | 1.2% | 1.4% | 0.9% | 0.8% | 0.6% | 0.7% | 0.2% | 0.2% |
| $\lambda_s=0.87055$ | $\lambda_s=0.75786$ | -0.1% | 0.2% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |
| | $\lambda_s=0.91172$ | 0.1% | -0.1% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% |

*In the individual lines all parameters are the same as in the baseline except the parameter indicated.*
*(For the values of 0.87055; 0.75786 and 0.91722 for the corresponding half-lives are 5; 2.5 and 7.5 years.)*

# 5 Summary and Assessment

Fiscal discipline has a large role in the real convergence process of Central and East European Countries. Fiscal policy aimed at cutting indebtedness by restraining consumption creates higher macroeconomic stability and an incentive for investors. This attracts foreign investment in a period when this increases productivity through external effects. Estimates of the model show that additional saving may bring an 18 percent return for consumers.

This rate is higher than any reasonable market rate but this is not surprising when the external effects incorporated into the model are considered. The question is whether this rate is consistent with the preferences of society? Is it reasonable for the government not to decide to increase savings if additional saving brings an 18 percent real yield?

We know that the social utility function is not the sum of individual utility functions. Part of the population has a very high time preference,[23] resulting in a behavior that considers savings a buffer stock against short-term income losses at best. Some of these “liquidity constrained” consumers are willing to take loans at a real rate even higher than 18 percent. These consumers might be satisfied with a policy strategy that reallocates future consumption into the present at an opportunity cost of 18 percent. However, whether such a policy should be seen as serving social welfare is questionable. Namely, there are many who would consider a borrowing at an 18 percent interest unreasonable. However, as this interest rate is not internalized for them, they have no direct free choice of making such a return by saving.

In this paper we did not discuss how the government could internalize this return. We set it as our task only to calculate the magnitude of a return that would allow policy makers to make choices on the basis of correct information.

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[23] *The idea of a high rate of time preference had come up as early as in Friedman (1957), and Carroll (1992) was one of those who contributed most to the rigorous analysis of the consequences. Most empirical econometric models use the related concept of Mankiw, who perceives the majority of consumers to be liquidity constrained, consuming as much as they earn. (The fiscal implications of this behavior are discussed in Mankiw (2000).)*

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# Appendix A: Derivation of the Parameters in the Consumption Function

In the $c_t = \exists_w w_{t-1} /(1+g_t) + \exists_y y_t^{lab}$ consumption function $\exists_w$ and $\exists_y$ have been determined by the constraint that consumption should converge to a $\overline{w}$ steady state wealth ratio with $0$ half-life. Let us first write the equation for $w_t$ wealth accumulation, using $k_{d,t} = \frac{1-\delta}{1+g_t} k_{d,t-1} + i_{d,t}$ and $nfa_t = \frac{(1+r_t^d)nfa_{t-1} - \pi_{f,t} k_{f,t-1}}{1+g_t} + tr_t + i_{f,t}$ equations.[24] Thus:

$$nfa_t + k_{d,t} = \frac{(1+r_t^d)nfa_{t-1} - \pi_{f,t} k_{f,t-1}}{1+g_t} + tr_t + i_{f,t} + \frac{1-\delta}{1+g_t} k_{d,t-1} + i_{d,t},$$

and by using definitions $tr_t = 1 - c_t - i_{f,t} - i_{d,t}$ and $y_t^{lab} = 1 - \frac{\pi_{f,t} k_{f,t-1} + \pi_{d,t} k_{d,t-1}}{1+g_t}$ – equations (D11) and (D19) –, after rearranging:

$$nfa_t + k_{d,t} = \frac{\left(1+r_t^d\right)nfa_{t-1} + (1+\pi_{d,t} - \delta)k_{d,t-1}}{1+g_t} + \left[y_t^{lab} - c_t\right],$$

and by substituting $r_t^{pf}$ and the definition of the wealth ratio ($w_t = nfa_t + k_{d,t}$) we get

$$w_t = \frac{1+r_t^{pf}}{1+g_t} w_{t-1} + \left[y_t^{lab} - c_t\right].$$

The consumption block in a narrow sense is a difference equation system for $c_t$ and $w_t$:

$$c_t = \beta_w \frac{w_{t-1}}{1+g_t} + \beta_y y_t^{lab}$$

$$w_t = \frac{1+r_t^{pf}}{1+g_t} w_{t-1} + \left[y_t^{lab} - c_t\right] .$$

After substituting $c_t$ into the second equation and rearranging:

(F1)
$$w_t = \frac{1+r_t^{pf} - \beta_w}{1+g_t} w_{t-1} + \left(1-\beta_y\right) y_t^{lab}.$$

Disregarding from the interaction among $y_t^{lab}$, $g_t$, $r_t^{pf}$ and $w_t$ around steady state the half-life of the differential equation is determined by $\frac{1+\overline{r}^{pf} - \beta_w}{1+\overline{g}}$. If we want to calibrate the model to a given $0$ half life, then by determining $\exists_w$ from $\frac{1+\overline{r}^{pf} - \beta_w}{1+\overline{g}} = \exp\left(\frac{\ln 0.5}{\eta}\right)$ we

[24] *See equations (D6) and (D16).*

get the (P1) formulation. From a given $\overline{w}$ we can express $\exists_y$ from equation (F1), because $\overline{w} = \exp\left(\frac{\ln 0.5}{\eta}\right)\overline{w} + \left(1-\beta_y\right)\overline{y}^{lab}$, and by rearranging we get the form (P2) that determines $\exists_y$.

## Appendix B: Derivation of the Equations in the Real Exchange Rate Block

We give here the derivation of equation (D28) that determines the $b_t$ changing variable. Let us write $q_{BS,t}$ everywhere instead of $q_t$ and replace $c_{ntr,t} + i_{ntr,t}$ in equation (D22) by the accounting identity (D27), and substitute the budget constraint of consumption and investment (D21) for $c_{tr,t} + i_{tr,t}$:

(D30)
$$q_{Bs,t} = \frac{A_{ntr}}{A_{tr}}\left(\frac{y_{ntr,t}}{c_t + i_t - q_{BS,t}y_{ntr,t}}\right)^{-\beta_d - 1}.$$

Rearranging for $y_{ntr,t}$ and using $c_t + i_t = 1 - tr_t$:

(D31)
$$y_{ntr,t} = \frac{1 - tr_t}{\left(\frac{A_{tr}}{A_{ntr}}q_{Bs,t}\right)^{\frac{1}{\beta_d + 1}} + q_{BS,t}}.$$

In equation (D26) by using equation (D25) we write $1 - q_{BS,t}\, y_{ntr,t}$ instead of $y_{tr,t}$ and plugging in for $y_{ntr,t}$:

(D32)
$$q_{BS,t} = b_t\left(\frac{\frac{1-tr_t}{\left(\frac{A_{tr}}{A_{ntr}}q_{Bs,t}\right)^{\frac{1}{\beta_d+1}} + q_{BS,t}}}{1 - q_{BS,t}\frac{1-tr_t}{\left(\frac{A_{tr}}{A_{ntr}}q_{Bs,t}\right)^{\frac{1}{\beta_d+1}} + q_{BS,t}}}\right)^{-\beta_s - 1}.$$

Expressing $b_t$ and rearranging we get the formula (D28).